Non-Classical Parsing

Special Course at the
Eötvös Loránd University

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Universität Potsdam
Institut für Informatik und Computational Science

Teaching at Universities

- Mathematics, mainly Algebra
- Theoretical Computer Science
- Logic
- Programming
- Software Engineering
- Compiler (and Program Transformation)
- several special courses, (automata, formal languages, information theory, ...)

- This course is financially supported by the Erasmus program of the EU.
- Contracts with the University of Potsdam exist.
- It is also open for students. You may spend a term or two in Potsdam.

Potsdam, the center of Prussian kings, is a direct neighbour of the city of Berlin.

Outline

1. Non-context-free phenomena
2. Non-context-free descriptors
3. Efficient parsing algorithms for non-context-free mechanisms (for CD grammar systems)
4. Summary

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Non-Context-Free Phenomena

- Programming languages
- Linguistics
- Developmental biology
- Molecular genetics
- Logic (*language of tautologies*)
- Economic modeling (*workflows*)
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Programming Languages: Static Semantic Constraints

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- Assume L_{Java} be context-free.
- Further consider the following regular language R:

```
class A {
    int x(0|1)*;

    public static void main(String[] args) {
        System.out.println(x(0|1)*);
    }
}
```

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- However, $h(L_{\text{Java}} \cap R) = \{ ww \mid w \in \{0,1\}^* \}$
  is not context-free, a contradiction.

Linguistics: Swiss German Word Order

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  John said that we Hans the house helped paint.
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- Jan säit das mer $(\text{d'chind})^i$ $(\text{em Hans})^j$ es huus haend wele $(\text{laa})^i$ $(\text{hälfe})^j$ aastriiche.
  John said that we $(\text{the children})^i$ $(\text{Hans})^j$ the house have wanted to $(\text{let})^i$ $(\text{help})^j$ paint.
  John said that we wanted to let Mary help Hans, Frank help Jessica, Chris help Lucy, and Vanessa help René paint the house.

Linguistics: Swiss German Word Order

- Mapping:
  - accusative case objects (d'chind) to a,
  - dative case objects (Hans) to b,
  - verbs requiring accusative case (laa), to c,
  - verbs requiring dative case (hälfe), to d,
  - erase everything else,

  Result: $a^i b^j c^i d^j$ for all sentences of this form:

  $\{ a^i b^j c^i d^j \mid i \geq 1, j \geq 1 \}$

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- If the verbs requiring accusative and dative case are mapped to a and b, respectively, then a subset of

  $\{ ww \mid w \in \{a, b\}^+,\ |w| \geq 2 \}$ is obtained.

Developmental Biology: Cell Division

- Growth/Development of organisms (cell division):

$$A \rightarrow BB$$

- Performed (almost) in *parallel*
- Sentential form $A \quad A \quad A \quad A$
  should rewrite to $BB\,BB\,BB\,BB$
- Non-context-freeness due to exponential growth

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  In principle yes, but those mechanisms are not feasible.

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  $\hookrightarrow$ **Different approaches?!**

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- Grammar systems
  Cooperation of several context-free grammars

Controlled Derivations—Example

Matrix grammars [Ábrahám (1965)]

- Line up rules to finite sequences, e.g.:

  $(S \rightarrow AB),\ (A \rightarrow aAb,\ B \rightarrow cB),\ (A \rightarrow ab,\ B \rightarrow c)\ \rightarrow\ \{\, a^n b^n c^n \mid n \geq 1 \,\}$

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- **Appearance checking (ac):** set of occurrences of rules that can be left out if not applicable to the sentential form (*here: empty*)

Parallel Derivations—Lindenmayer systems

- Modeling of biological developmental processes [Lindenmayer 68], ...
- All symbols can be rewritten
- Parallel replacements of all symbols
- *Example:* $\{ a^{2^n} \mid n \geq 0 \}$ with $a \to a^2$

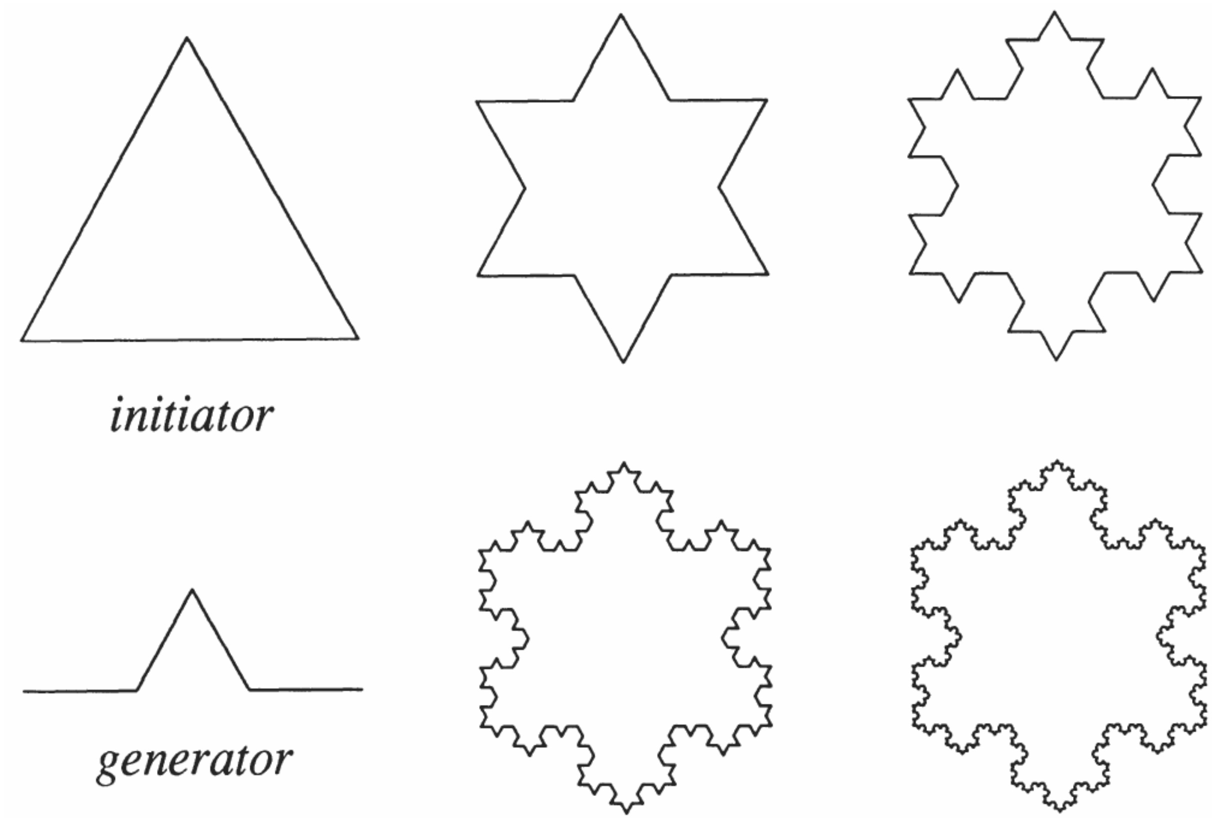
$$a \Longrightarrow aa \Longrightarrow aaaa \Longrightarrow a^8 \Longrightarrow a^{16} \Longrightarrow \ldots$$

Selected Applications

Reference:

Przemyslaw Prusinkiewicz, Aristid Lindenmayer,
The Algorithmic Beauty of Plants, Springer 1990.

Fractals

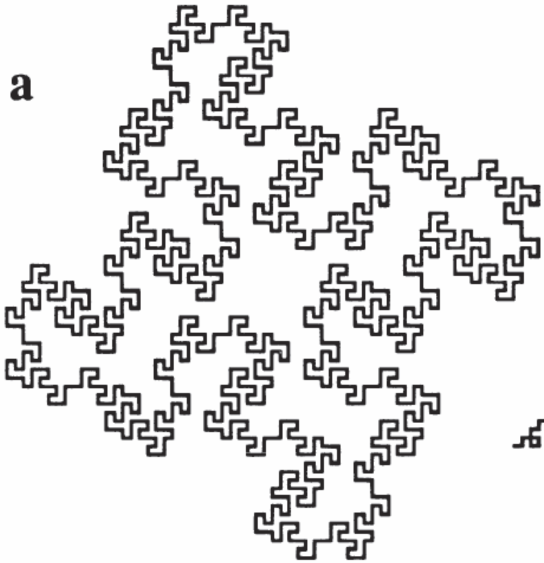


Turtle Graphic

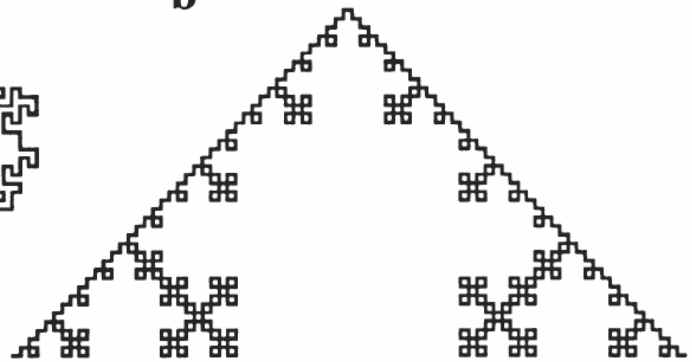
α, δ	two angles
α	initial angle with x-axis
F	draw line of unit length "'straightforward"'
+	change direction by $+\delta$
-	change direction by $-\delta$

Fractals (2)

a



b

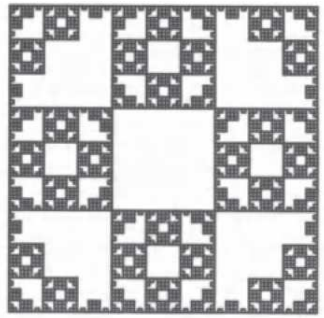


$n = 2,\ \delta = 90°$
F-F-F-F
F → F+FF-FF-F-F+F+F
F-F-F+F+FF+FF-F

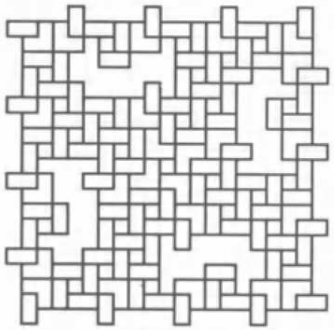
$n = 4,\ \delta = 90°$
-F
F → F+F-F-F+F

Fractals (3)

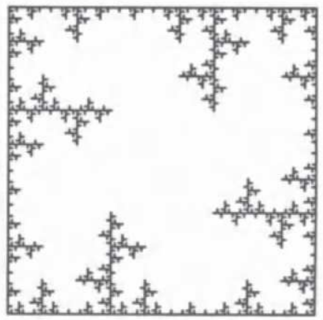
a n = 4, δ = 90°
F-F-F-F
F → FF-F-F-F-F-F+F



b n = 4, δ = 90°
F-F-F-F
F → FF-F-F-F-FF

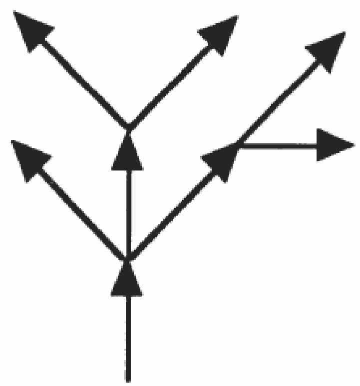


c n = 3, δ = 90°
F-F-F-F
F → FF-F+F-F-FF



d n = 4, δ = 90°
F-F-F-F
F → FF-F--F-F

Graphical Interpretation with Stack Operations



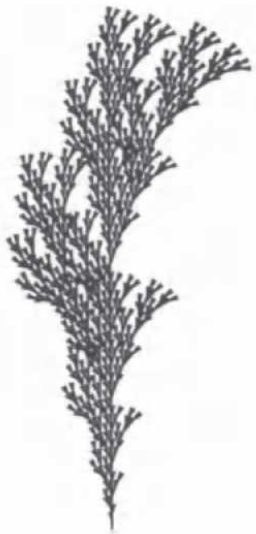
F[+F][-F[-F]F]F[+F][-F]

[push turtle state onto stack

] pop state from stack and set turtle to this state
(by moving turtle without drawing a line)

Branching Structures (1)

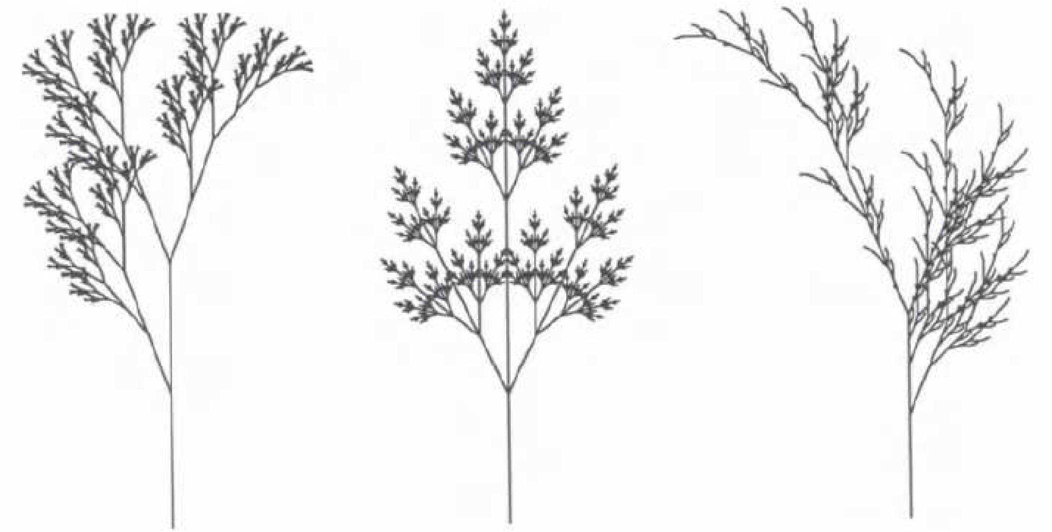
a
n=5, δ=25.7°
F
F $\rightarrow$ F[+F]F[-F]F



b
n=5, δ=20°
F
F $\rightarrow$ F[+F]F[-F][F]

c
n=4, δ=22.5°
F
F $\rightarrow$ FF-[-F+F+F]+
[+F-F-F]

Branching Structures (2)



d
n=7, δ=20°
X
X →F[+X]F[-X]+X
F →FF

e
n=7, δ=25.7°
X
X→F[+X][-X]FX
F→FF

f
n=5, δ=22.5°
X
X→F-[[X]+X]+F[+FX]-X
F→FF

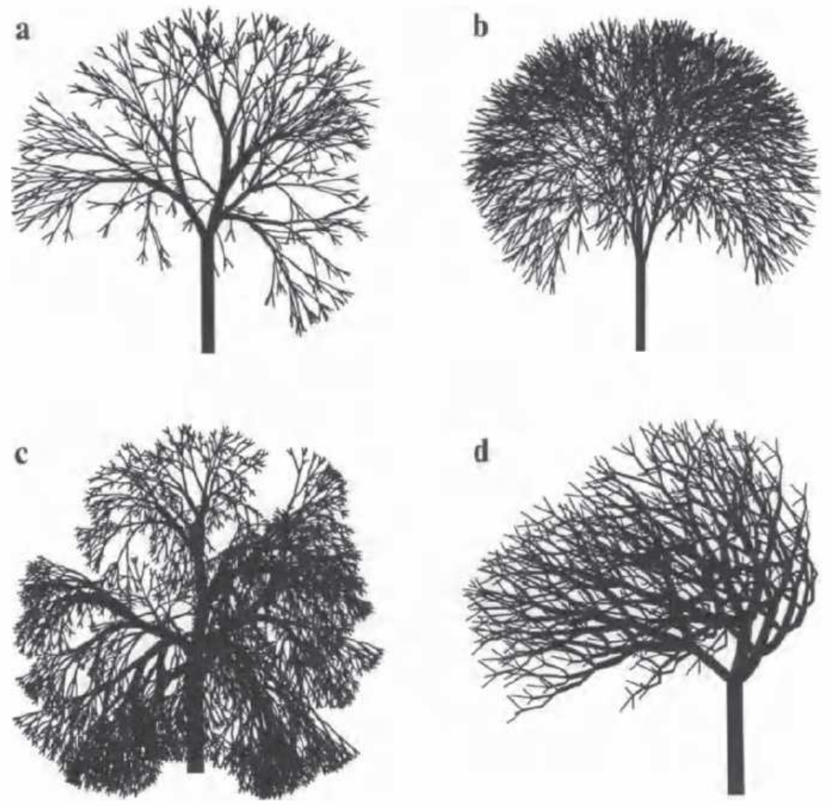
Three-Dimensional Graphics with Textures

n=7, δ=22.5°

ω : A
p_1 : A → [&FL!A]/////'[&FL!A]///////'[&FL!A]
p_2 : F → S ///// F
p_3 : S → F L
p_4 : L → ['''∧∧{-f+f+f-|-f+f+f}]

Three-Dimensional Graphics with Textures (2)

Parametrized Descriptions




```
#define d1 94.74       /* divergence angle 1 */
#define d2 132.63      /* divergence angle 2 */
#define a 18.95        /* branching angle */
#define lr 1.109       /* elongation rate */
#define vr 1.732       /* width increase rate */
```

ω : `!(1)F(200)/(45)A`

p_1 : `A : *` $\rightarrow$ `!(vr)F(50)[&(a)F(50)A]/(d1)`
`[&(a)F(50)A]/(d2)[&(a)F(50)A]`

p_2 : `F(l) : *` $\rightarrow$ `F(l*lr)`

p_3 : `!(w) : *` $\rightarrow$ `!(w*vr)`

Lindenmayer Systems: Variants

Determinism	exactly one rule per symbol $\hookrightarrow$ iterated homomorphisms
Tables	several rule sets
Extension	auxiliary symbols
Adult	ruling out all fixed points
...	

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ET0L systems — nondeterministic extended tabled L systems

0: applicability of rules depends on zero neighbouring symbols (context-free derivation)

Grammar Systems (GS)

- *Idea:* Several context-free grammars (*components*) jointly generate the strings of the language.

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- CD-GS: *Sequential cooperation* [Csuhaj-Várju, Dassow (1992)] working on a common sentential form in turns
  - Distributed problem solving in blackboard architectures
  - Multi-level grammars [Meersman, Rozenberg (1978)]
  - Sequential analogue to tabled Lindenmayer systems [Bordihn, Csuhaj-Varjú, Dassow (1997)]

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- PC-GS: *Parallel cooperation* [Păun, Sântean (1989)] autonomous, synchronized derivations and communication of sentential forms upon request (by particular nonterminal symbols)

CD Grammar Systems—Definition

- A context-free CDGS of degree n is a tuple

$$\Gamma = (N, T, S, P_1, P_2, \ldots, P_n)$$

  - N is a finite set of nonterminal symbols,
  - T is a finite set of terminal symbols
  - $S \in N$ (axiom),
  - P_i $(1 \leq i \leq n)$ is a finite set of context-free productions of the form $A \to \alpha$, $A \in N$, $\alpha \in (N \cup T)^*$

$\hookrightarrow$ Each (N, T, S, P_i) $(1 \leq i \leq n)$ is a context-free grammar (component).

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- $x \Longrightarrow_i y$ iff $\quad x = \gamma_1 A \gamma_2, \quad y = \gamma_1 \alpha \gamma_2, \quad A \rightarrow \alpha \in P_i$

Cooperation Strategies

- Components work sequentially (in turns) on a common sentential form
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Derivation mode	Number of steps to be performed (of a component, once activated)
$*$-mode	arbitrary
$=m$-mode	exactly m
$\leq m$-mode	at most m
$\geq m$-mode	at least m
t-mode	as many as possible
full-mode	until a nonterminal has been introduced that the component cannot replace

- $x \overset{=m}{\Longrightarrow}_i y$ iff $x = x_0 \Longrightarrow_i x_1 \Longrightarrow_i \cdots \Longrightarrow_i x_m = y$

- $x \overset{\mathrm{t}}{\Longrightarrow}_i y$ iff $x \overset{*}{\Longrightarrow}_i y$ and there is no z such that $y \Longrightarrow_i z$

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- For $\mu \in \{ =m, \mathrm{t} \mid m \geq 1 \}$:

  $$L(\Gamma, \mu) = \{ w \in T^* \mid S \stackrel{\mu}{\Longrightarrow}_{i_1} v_1 \stackrel{\mu}{\Longrightarrow}_{i_2} \ldots \stackrel{\mu}{\Longrightarrow}_{i_\ell} w, \ \ell \geq 1,\ 1 \leq i_j \leq n \text{ for } 1 \leq j \leq \ell \}$$

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Example:

$P_1 = \{S \to S,\ S \to AB\}$

$P_2 = \{A \to aA',\ B \to cB'\}$ $P_3 = \{A \to A'b,\ B \to B'd\}$

$P_4 = \{A' \to A,\ B' \to B\}$ $P_5 = \{A \to \lambda,\ B \to \lambda\}$

$$L(\Gamma, =2) = L(\Gamma, \mathrm{t}) = \{ a^i b^j c^i d^j \mid i, j \geq 0 \}$$

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The Goal

- **Restricting** CDGS (in t- and $=m$-modes) such that
  - efficient top-down parsing becomes possible
    $\hookrightarrow$ deterministic one-way parsing without backtracking (*here:* top-down),
  - important non-context-free languages can be generated

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$*$ Removing nondeterminism:

- Leftmost derivations
- $\mathrm{LL}(k)$-condition

Leftmost Derivations

- strong leftmost mode ($x \overset{\mu}{\underset{\mathrm{s}}{\Longrightarrow}}_i y$):
  always replace the leftmost nonterminal in the sentential form

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- weak leftmost mode ($x \overset{\mu}{\underset{\mathrm{w}}{\Longrightarrow}}_i y$):
  - When no component has been activated yet (in the first step of some P_i): always replace the leftmost nonterminal in the sentential form
  - If the component P_i is already active: always replace the leftmost symbol in the sentential form that is from the domain $\mathrm{dom}(P_i)$

LL(k) condition for CDGS

Given: 1) CDGS Γ, $\gamma \in \{\mathrm{s}, \mathrm{w}\}$, $\mu \in \{\,\mathrm{t}, =m \mid m \geq 1\,\}$

2) Input to be analyzed $w = a_1 a_2 \ldots a_s \in T^*$

Question: Does $w \in L_\gamma(\Gamma, \mu)$ hold?

Goal: Re-construction of a leftmost derivation leading from S to w if $w \in L_\gamma(\Gamma, \mu)$

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Let $S \overset{\mu}{\underset{\gamma}{\Longrightarrow}}_{i_1} \ldots \overset{\mu}{\underset{\gamma}{\Longrightarrow}}_{i_j} a_1 a_2 \ldots a_r A y$ be already analyzed ($y \in V^*$).

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LL(k) condition: Then the next k input symbols (tokens)
$a_{r+1} \dots a_{r+k}$ (look-ahead)
determine the unique next derivation step $\overset{\mu}{\underset{\gamma}{\Longrightarrow}}_{i_{j+1}}$

Context-Free $\mathrm{LL}(k)$ Parsing

Is using look-ahead $k > 1$ an issue?

- $\mathrm{LL}(1) \subset \mathrm{LL}(2) \subset \mathrm{LL}(3) \subset \ldots$

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$$Var \to SimpleVar \mid IndexedVar$$
$$SimpleVar \to \mathrm{Idf}$$
$$IndexedVar \to \mathrm{Idf}[Expr]$$

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$$\begin{aligned}
&Var \rightarrow SimpleVar \mid IndexedVar\\
&SimpleVar \rightarrow \mathrm{Idf}\\
&IndexedVar \rightarrow \mathrm{Idf}[Expr]
\end{aligned}$$

- Var-rule requires look-ahead of size 2
- Can hardly be improved since $SimpleVar$ is likely to be used in many other rules

$\mathbf{LL}(k)$ Parsing Tables

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- Unfortunately, this works only for **strong** $\mathrm{LL}(k)$ grammars.
- Every $\mathrm{LL}(1)$ grammar is strong $\mathrm{LL}(1)$, but strongness is a restriction for $\mathrm{LL}(k)$ grammars if $k > 1$.

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- Construction of strong $\mathrm{LL}(2)$ tables disregards "history" of derivation

- Can be fixed by using distinct copies of A_1 and A_2 of A

Parsing Tables for CDGS

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- **Simplification:** only the label of the component is needed,
  if the CDGS is deterministic, i.e., each P_i contains at most one rule for every nonterminal symbol.

Example

$$P_1 = \{S \to S,\ S \to AB\}$$
$$P_2 = \{A \to aA',\ B \to cB'\} \qquad P_3 = \{A \to A'b,\ B \to B'd\}$$
$$P_4 = \{A' \to A,\ B' \to B\} \qquad P_5 = \{A \to \lambda,\ B \to \lambda\}$$

$$L_{\mathrm{w}}(\Gamma, =2) = L_{\mathrm{w}}(\Gamma, \mathrm{t}) = \{\, a^i b^j c^i d^j \mid i, j \geq 0 \,\}$$

	a	b	c	d	λ
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A	2	3	5	5	5
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$\hookrightarrow$ satisfies the condition for $\mathrm{LL}(1)$ CDGS

Computational Power of $\mathrm{LL}(k)$ CDGS

[Bordihn, Vaszil 2007]

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- One can describe:
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  - $\{\, a^i b^j c^i d^j \mid i,j \geq 0 \,\}$, $\{\, wcw \mid w \in \{a,b\}^* \,\}$, $\{\, a^i b^i c^i \mid i \geq 0 \,\}$ (using weak leftmost derivations)
  - languages that are not semi-linear (using weak leftmost derivations)

Parsing Algorithm—Idea

- **Weak leftmost derivations:** symbols have to be replaced that may appear far away from the leftmost nonterminal.

$$S \overset{=2}{\underset{\mathrm{w}}{\Longrightarrow}}_1 AB \overset{=2}{\underset{\mathrm{w}}{\Longrightarrow}}_2 aAcB \overset{=2}{\underset{\mathrm{w}}{\Longrightarrow}}_2 \ldots \overset{=2}{\underset{\mathrm{w}}{\Longrightarrow}}_2 a^r Ac^r B \ldots$$

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- Control usage of productions stored in queues with the help of stacks that, for every occurrence of a nonterminal in the sentential form, store the time (derivation step) when it has been generated.

Parsing Algorithm—Result

Theorem. Let Γ be a CDGS working in $=m$-mode $(m \geq 2)$ with weak leftmost derivaions, and let Γ satisfy the $\mathrm{LL}(k)$ condition for CDGS $(k \geq 1)$.

Given the parsing table for Γ, one can effectively construct a parser for $L_{\mathrm{w}}(\Gamma, =m)$.

For every input string, the parser terminates in $O(n \cdot \log^2 n)$ time, where n is the length of the input.

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- All relevant CDGS are strong $\mathrm{LL}(k)$.

Outline

1. Non-context-free phenomena
2. Non-context-free descriptors
3. Accepting grammars
4. Efficient parsing algorithms for non-context-free mechanisms (for CD grammar systems)
5. Summary

Summary

- $\mathrm{LL}(k)$ CDGS describe all "classical" context-free $\mathrm{LL}(k)$ languages as well as crucial non-context-free languages.
- $\mathrm{LL}(k)$ CDGS have an efficient parser (with $O(n \cdot \log^2 n)$ time complexity in the worst case).
- The $\mathrm{LL}(k)$-hierarchy collapses for CDGS at the first level. (Condition: Parsing table is given!)
- If a CDGS is strong-$\mathrm{LL}(k)$, then its parsing table can effectively be constructed. All relevant examples are strong-$\mathrm{LL}(k)$.